Program With Abstracts – Midwest PhilMath Workshop 2022
(updated 10/24)
Lectures will take place in Room 101, Jordan Hall of Science,
University of Notre Dame

Saturday, October 29

9:00 am **Gabriel Day**: A New Argument against Non-Well-Founded Set Theory

Non-well-founded set theory, which allows for sets with loops and infinitely descending chains of membership, has long been excluded from mainstream logical and philosophical discourse. In spite of a widespread attitude that such sets are too strange to exist, there is a tradition going back at least to Peter Aczel's work which formalizes non-well-founded sets using certain kinds of graphs. In this paper, I argue that anyone who wishes to use these graph-based set theories to demonstrate metaphysical points about sets— for instance, that self-membered sets really exist—must accept the graph conception of set. This principle, formulated by Luca Incurvati,asserts that sets are the things depicted by graphs. I then examine two possible reasons to jettison the graph conception. The first is the claim that graphs cannot be coherently defined without reference to sets; this idea is undermined by an axiomatic approach to graph theory. Nevertheless, the graph conception is mistaken, because it artificially reduces sets to graphs, violating mathematical practice in much the same way as set-theoretic reductionism. Hence the graph-based set theories introduced by Aczel should be rejected as descriptions of the ontology of sets.

10:10 am **Matteo de Ceglie**: What's a Multiverse? Generalising on the notion of a Set-Theoretic Multiverse.

10 years ago, Hamkins (2012) introduced the notion of set-theoretic multiverse to better account for current set-theoretic practice (and especially our intuitions surrounding the application of forcing). After his seminal paper, research on the set-theoretic multiverse focussed on two main topics: the debate between universism (the belief that there exists only one, unique set-theoretic multiverse) and multiversism (according to which there are several, all equally legitimate, set-theoretic universes); and the introduction of new characterisations of the multiverse. The latter gave rise to multiverses that are both philosophically interesting and mathematically fruitful. However, these multiverses are each investigated in isolation, and there is still no unified theory, or definition, of the "set-theoretic multiverse". Current set-theoretic practice is filled with results that start with "Let M be a model of ZFC...", but there is still no general result about the set-theoretic multiverses, i.e. "Let M be a multiverse of set theory...". In this talk, I plan to take the first steps towards a general theory of the multiverse. To do so, consider the collection of all models of ZFC, Mod(ZFC). On this collection we can define a binary relation R, such that R(M,N) <-> N is an extension of M (e.g. N could be a set-generic extension of M, or a class-generic one, etc.). By taking the closure R* of R by adding all grounds (i.e. every x such that R(x,M)) and all extensions (i.e. every y such that R(M,y)), we can carve the collection Mod(ZFC) in non-overlapping partitions of models of ZFC. Each one of these partitions is a generic multiverse. If we then change the definition of R, e.g. by admitting only set-generic or class-generic extensions, or top/end-extensions, etc., when we take the corresponding closure we end up defining different multiverses on Mod(ZFC). In this talk I will expand on the

behaviour of this relation R, and discuss the consequences of considering different collections of models (e.g. Mod(ZF)), different possible definitions of R, and different ways to take the closure of R.

11:20 am **Ed Zalta**: Mathematical Pluralism

Mathematical pluralism can take one of three forms: (1) every consistent mathematical theory consists of truths about its own domain of individuals and relations; (2) every mathematical theory, consistent or inconsistent, consists of truths about its own (possibly uninteresting) domain of individuals and relations; and (3) the principal philosophies of mathematics are each based upon an insight or truth about the nature of mathematics that can be validated. (1) includes the multiverse approach to set theory. (2) helps us to understand the significance of the distinguished non-logical individual and relation terms of even inconsistent theories. (3) is a metaphilosophical form of mathematical pluralism and hasn't been discussed in the literature. In what follows, I show how the analysis of theoretical mathematics in object theory exhibits all three forms of mathematical pluralism.

12:30 pm - Lunch

2:30 pm **Susan Sterrett** How mathematics figures differently in Exact Solutions, Simulations, and Physical Models

The role of mathematics in scientific practice is too readily relegated to that of formulating equations that model or describe what is being investigated, and then finding solutions to those equations. I survey the role of mathematics in: 1. Exact solutions of differential equations, especially conformal mapping; and 2. Simulations of solutions to differential equations via numerical methods and via agent-based models; and 3. The use of experimental models to solve equations (a) via physical analogies based on similarity of the form of the equations, such as Prandtl's soap-film method, and (b) the method of physically similar systems.

Two major themes emerge: First, the role of mathematics in science is not well described by deduction from axioms, although it generally involves deductive reasoning. Creative leaps, the integration of experimental or observational evidence, synthesis of ideas from different areas of mathematics, and insight regarding analogous forms are required to find solutions to equations. Second, methods that involve mappings or transformations are in use in disparate contexts, from the purely mathematical context of conformal mapping where it is mathematical objects that are mapped, to the use of concrete physical experimental models, where one concrete thing is shown to correspond to another.

3:45 pm **Ellen Lehet and William D'Alessandro**: Mathematical Explanation and Understanding: A Noetic Account

We defend a noetic account of intramathematical explanation. On this view, a piece of mathematics is explanatory just in case it produces an appropriate type of understanding. We motivate the view by presenting some appealing features of noeticism. We then discuss and criticize the most prominent extant version of noeticism, due to Matthew Inglis and Juan-Pablo Mejía-Ramos, which identifies explanatory understanding with the possession of detailed cognitive schemas. Finally, we present a novel noetic account. On our view, explanatory understanding arises from meeting specific explanatory objectives, the theory of which we briefly set out.

5:00 pm **Jeffrey Schatz and Sorin Bangu**: Full-blooded Intensionalism in Mathematics
This talk has two goals. First, we reconstruct Wittgenstein's views on what counts as a legitimate irrational -- since, as he repeatedly points out, and in agreement with mathematicians such as Emile Borel, not just every infinite string of digits qualifies as one. Once his conception ('full-blooded intensionalism') is sketched out, and its specificity highlighted by comparing it with two other cognate views ('extensionalism' and 'quasi-intensionalism'), our second objective is to examine how his type of intensionalism impacts his attitude towards Cantor's theorem. In this regard, the more general claim we argue for is that, despite appearances to the contrary, Wittgenstein was not a revisionist about set-theoretical practice.

6:45 pm: Reception and Dinner

Sunday, October 30
9:00 am **John Baldwin and Andreas Mueller**: What is a Geometric Proof? The Dilemma of de Zolt's Axiom
We explore the notion of geometric proof by distinguishing between two approaches: a) giving (showing the existence of) a formal proof of a first order statement from geometric axioms and b) the usual mathematical approach: proving a geometric result in a base theory such as ZFC. De Zolt's axiom attempts to provide a rigorous base for Euclid's proof that polygons can be linearly ordered by area. It relies on Euclid's definition of equal area which is formalized in $L_{\omega_1,\omega}$. Hilbert proves De Zolt's axiom from his first order axioms for geometry using a measure of area function. After Gödel and Tarski, we can see that this result uses approach b). In the process he proved the first order interdefinability of Euclidean geometries with the theory of fields. But Euclid's definition is not first order. Refining Hilbert's claim that his argument is geometric (e.g. [Hil71], p. 64]), Hartshorne ([Har00], 23.6.1) asks whether there is a geometric proof that avoids `measure of area'. We explore approach b) by embedding Hartshorne's question about the comparison of area in first order Euclidean plane geometry into the problem of `comparing magnitudes' in a more general context including non-Archimedean geometries, higher dimensions, and non-Euclidean geometries. In this more general context, we are able to sharpen Hartshorne's objection and examine Hilbert's implicit use of number in a way that was anathema to Euclid.

References:

[Har00] Robin Hartshorne. *Geometry: Euclid and Beyond*. Springer-Verlag 2000
[Hil71] David Hilbert. *Foundations of Geometry*. Open Court 1971; translation from the 10th German edition by Harry Gosheen, edited by Bernays 1968.

10:10 am **Guillaume Massas**: A Semi-Constructive Approach to the Hyperreal Line
Although nonstandard analysis has grown into a diverse field that studies a wide variety of structures, many of its applications to ordinary mathematics rely on the existence of a hyperreal line, i.e., a first-order structure M satisfying versions of the following axioms [1]:
1. (Extension Principle) The signature of M contains a nonstandard extension S∗ for any

finitary relation S on R.
2. (Transfer Principle) R is an elementary substructure of M.
3. (Saturation Principle) Any countable sequence of non-empty nested internal subsets of R∗ has a non-empty intersection.
It is well-known that the existence of a nonstandard extension satisfying these three conditions implies the existence of a non-principal ultrafilter on ω and thus exceeds the resources of semi-constructive mathematics (ZF +DC), a natural setting for standard analysis [6, Chap. 14]. In this talk based on [4], I will present a way out of this problem that relies on an alternative to Tarskian semantics known as possibility semantics [2]. In particular, I show that the appeal to classical ultrapowers modulo a non-principal ultrafilter on ω can be avoided by using the partially-ordered set of all non-principal filters on ω. This allows for the definition of a semiconstructive analogue †R of the hyperreal line which satisfies versions of the Extension, Transfer and Saturation Principles and can serve as an alternative foundation for infinitesimal calculus `a la Keisler [3].
I will compare my proposal to some other alternative approaches to nonstandard analysis, including Tao's cheap nonstandard analysis [8], Palmgren's sheaf-theoretic approach [5] and Scott's Boolean-valued analysis [7], and I will conclude by discussing the relevance of this new proposal for some recent debates regarding the applicability of nonstandard methods to ordinary mathematics.

11:20 am **Katalin Bimbó**: Proofs – or something like that

Mathematical knowledge is justified by proofs. Ideally, proofs are constructed in a formal system, perhaps, in an axiomatic theory modeled after Euclid's presentation of geometry. The crisis in the foundations of mathematics prompted renewed concerns about proofs, most prominently, in formalist approaches to mathematics. Nevertheless, even today, there is a substantial discrepancy between proofs carried out in formal systems (e.g., ZFC or NBG) and proofs presented by practicing mathematicians in texts.
In this talk, I will focus on proofs that are fully formalized and computerized. There are known concerns that can emerge about such proofs. For instance, the resulting software may contain coding mistakes, and the size of the proof may defy verification by humans. A further source of problems is the translation of informal steps into formal ones, alternatively, the interpretation of formal steps in colloquial mathematical language. A potential conclusion to be drawn from these considerations is that augmenting logical frameworks may be a more promising approach to close the gap between informal and fully formalized proofs.

12:30 - *Adjournment*