Midwest PhilMath Workshop 2020 – University of Notre Dame
October 14 – December 9, 2020

Abstracts of Papers

Steve Awodey: "Intensionality, Invariance, and Univalence"

What does a mathematical proposition mean? Under one familiar account, all true mathematical statements mean the same thing, namely True. A more meaningful account is provided by the Propositions-As-Types conception of type theory, according to which the meaning of a proposition is its collection of proofs. The new system of Homotopy Type Theory provides a further refinement: The meaning of a proposition is the homotopy type of its proofs. A homotopy type may be seen as an infinite-dimensional structure, consisting of objects, isomorphisms, isomorphisms of isomorphisms, etc. Such structures represent systems of objects together with all of their higher symmetries. The language of Martin-Löf type theory is an invariant of all such higher symmetries, a fact which is enshrined in the celebrated Principle of Univalence.

Anna Bellomo and Guillaume Massas: "Bolzano's Mathematical Infinite"

Bernard Bolzano's *Paradoxien des Unendlichen* (*PU* for short) is one of his most widely read works. Even so, it is still a fascinatingly fresh read for the contemporary philosopher of maths, as its standing with respect to the broader history of mathematical infnity remains little understood (*cf.*, e.g., Berg 1962, 1992; ˆSebestík 1992; Rusnock 2000). This paper gives a new interpretation of Bolzano's "calculation of the infnite" as presented in *PU*, xx 29-33. We claim that, contrary to a widespread view (e.g. Berg 1962) Bolzano's treatment of the size of infinite (countable) collections is not a defective (Berg) or inconsistent (Sebestik) anticipation of Cantorian set theory, but rather a sketch of an alternative theory of the infinite. The key to understanding *PU* xx29-33, we argue, is realizing that Bolzano is comparing the size of infinite sums, not infinite sets or set-like collections. After carefully analyzing the relevant passages in the *PU*, we bolster our interpretation by modelling Bolzano's computations with infinite sums with an ultrapower construction. While ultrapowers have recently become central tools in developing non-Cantorian accounts of the size of infinite sets more generally (Benci and Di Nasso 2003; Trlifajová 2018), our specific construction allows us to reproduce Bolzano's more surprising results while remaining faithful to his own reasoning. This builds a strong case that Bolzano's views on the countable infinite can be read as being consistent yet different from Cantor's in a deep way.

Ethan Brauer: "A Modal theory of Free Choice Sequences"

In developing his intuitionistic theory of the continuum, L.E.J. Brouwer developed the notion of a free choice sequence, a potentially infinite sequence of numbers chosen more or less freely by the mathematical agent. Free choice sequences have been poorly received outside intuitionist circles, objected to on the grounds that they make mathematics subjective, they introduce a temporal aspect to mathematics, and they lead to results that contradict classical mathematics. Against these objections, I argue that a notion of potential infinity is sufficient to make sense of free choice sequences. Using recent modal accounts of potential infinity, I will develop a modal theory of free choice sequences over a background classical logic. Showing how this theory can capture many of the ideas and results of the intuitionistic theory of choice sequences thus defuses the objections made against choice sequences.

Walter Dean and Hidenori Kurokawa: "On the Methodology of Informal Rigour: Set Theory, Semantics, and Intuitionis""

Abstract coming soon

Geoffrey Hellman and Stewart Shapiro: A Classical-Modal Interpretation of Smooth Infinitesimal Analysis

A remarkable 20th C. development is smooth infinitesimal analysis (SIA), which replaces the method of limits of classical analysis (CA) in favor of a nilsquares object, Δ, which can be proved not to be = {0}, but which cannot be proved to contain any element $\varepsilon \neq 0$. To avoid contradiction, the logic of SIA must be intuitionistic (in which the quantifier conversion from '$\neg\forall$' to '$\exists\neg$' is not valid). Yet, unlike intuitionistic or Bishop analysis, not all axioms of SIA can be read constructively, including the key axiom of *microaffineness*, as will be explained. Thus, a philosophical challenge is posed, to motivate the restriction to intuitonist logic (beyond citing inconsistency resulting from classical logic). The key idea of our current work is to treat certain identity conditions, e.g. relating generic nilsquares and 0, as inherently indeterminate or 'vague'. Using 'D' as a modal operator, "it is determinate that", behaving like necessity in S4, we allow e.g. $\neg D(\varepsilon = 0) \wedge \neg D(\varepsilon \neq 0)$ to be satisfied at given worlds. Calling on the well-known Gödel translation of an intuitionistic theory into the language of S4, we have the determinateness of the SIA axioms as axioms of our CM theory. As Gödel proved, the translation is proof-theoretically faithful, establishing relative consistency. The remainder of our talk will describe an improvement of CM, deploying a 3-valued (otherwise classical) logic, and two extensions by different, mutually exclusive axioms, determinateness of positive identities, and determinateness of indeterminate identities of the form, $\varepsilon = 0$.

Daniel Isaacson: "Kreisel's Philosophy of Mathematics

Kreisel has described how his interest in foundations of mathematics arose early: "Since my school days I had had those interests in foundations that force themselves on beginners when they read Euclid's Elements (which was then still done at school in England), or later when they are introduced to the differential calculus." At the same time, he had a mathematician's distrust of philosophy of mathematics, though he was a philosopher of mathematics himself, in the way in which mathematicians such as Cantor, Dedekind, Hilbert, Brouwer, Weyl, and Gödel were philosophers of mathematics, motivating and justifying the way in which they did their mathematics. Among Kreisel's more than 200 publications, a relatively small number are explicitly philosophical, but these grow out of and at the same time inform the whole body of his work. Even when we recognize Kreisel as a philosopher of mathematics, it's not immediately easy to label his philosophy of mathematics. My talk will be a preliminary attempt to do this.

Graham Priest **"Perspectives on the Universe"**
Joel Hamkins has advanced a well known view to the effect that there is no unique universe of sets. There is simply a plurality of such universes. We have, then, a pluriverse. A natural objection to this view is that there is still a single universe: the totality, V, in which all the members of the pluriverse find themselves. In this paper I consider a reply to the objection, to the effect that there is no such thing as V in itself. Rather, each member of the pluriverse simply gives a different perspective on what V is. This view is then generalised in the light of mathematical pluralism. What emerges is a vastly expanded, and logic-neutral view of the pluriverse.

Giuseppina Ronzitti: "Intuitionism Without Intuition: Against the Phenomenological Account"
In this paper, we will consider the impact of the *intuitionistic philosophical program* on the *intuitionistic mathematical program*. In particular, we will concentrate on the phenomenological approach to the philosophy of intuitionism. We shall argue that recent attempts (such as that of Mark van Atten) to justify intuitionistic mathematics by appealing to Husserlian phenomenology are seriously contributing to the failure of the intuitionistic mathematical program. Our claim is that one of the main reasons for the failure of the intuitionistic mathematical program lies in the emphasis that is given to the philosophical program. Our thesis will be illustrated by an example: the phenomenological justification of the intuitionistic notion of choice sequence.
As a matter of fact, not much intuitionistic mathematics has been produced so far and, in this sense, we might say that the intuitionistic mathematical program is failing. Nevertheless, we think that the intuitionistic mathematical program might be defensible, but it should be defended directly, by the actual production of significant pieces of intuitionistic mathematics, rather than trying to legitimate its entities and principles of reasoning by appealing to philosophical theories.
It should be clear from the start that it is beyond the scope of this paper to criticize in detail the phenomenological approach to intuitionism. Our criticism is more of a methodological nature, and it amounts to saying that it does not appear to be a good strategy to require from mathematicians that they embrace phenomenology before actually doing intuitionistic mathematics.

Chris Scambler: "Can All Things Be Counted?"
I will present a modal axiom system for set theory that (I argue) reconciles mathematics after Cantor with the idea there is only one size of infinity. I'll begin with some philosophical background on Cantor's proof and its relation to Russell's paradox. I'll then show how techniques developed to treat Russell's paradox in modal set theory can be generalized to produce set theories consistent with the idea that there's only one size of infinity. I'll also motivate one such theory in terms of the idea that forcing should always be possible (over arbitrary partial orders), an idea which has seen some recent interest among philosophically inclined set theorists.

Will Stafford and Andrew Arana: "Discovermental Complexity and Genus"

An account of mathematical understanding should account for the differences between theorems whose proofs are “easy” to discover, and those whose proofs are difficult to discover. Though Hilbert seems to have created proof theory with the idea that it would address this kind of “discovermental complexity” (cf. Detlefsen 1990, p. 376n24), much more attention has been paid to the lengths of proofs, a measure of the difficulty of verifying of a given formal object that it is a proof of a given formula in a given formal system. In this paper we will shift attention back to discovermental complexity, by addressing a “topological” measure of proof complexity recently highlighted by Alessandra Carbone (2009). Though we will contend that Carbone’s measure fails as a measure of discovermental complexity, it forefronts numerous important formal and epistemological issues that we will discuss, including the structure of proofs and the question of whether impure proofs are systematically simpler than pure proofs.

Valérie Lynn Therrien: “On Counting as Mathematical Progress: Kuratowski-Zorn’s Lemma and the Path Not Taken”

In her *Naturalism in Mathematics*, Maddy claims that historical case studies give us sufficient reason to exclude extra-mathematical considerations from our account of mathematical progress. Indeed, she vouches that historical case studies can be tested against the predictions of a reconstructed means-end analysis. In this paper, we will take up this formidable challenge. We aim to do so *via* a carefully chosen case study designed to test the limits of a rational reconstruction’s ability to predict not only the path taken by mathematics, but also the path *not* taken by mathematics: the case of the Kuratowski-Zorn Lemma. Can Maddy’s framework account for why Zorn’s Lemma counts as mathematical progress, but Kuratowski’s prior equivalent maximal and minimal principle does not? While Maddy has done ground-breaking work in rationally reconstructing the path taken by set theory, it is not clear that her account can provide a convincing rationale for the path not taken. Our conclusion is that, while Maddy’s account provides a razor-thin margin of success, it also does not take into account salient extra-mathematical considerations. Ultimately, it is unlikely to be convincing to anyone not epistemologically committed to mathematical naturalism.