Summaries

MWPMW 13

University of Notre Dame
Department of Philosophy
October 27th–28th, 2012

Session I

• John Baldwin (Dept of Mathematics & Statistics, University of Illinois–Chicago): "Completeness and Categoricity (in power): Formalization without Foundationalism"

Summary

Formalization has three roles: 1) a foundation for an area (perhaps all) of mathematics, 2) a resource for investigating problems in 'normal' mathematics, 3) a tool to organize various mathematical areas so as to emphasize commonalities and differences.

We focus on the use of theories and syntactical properties of theories in roles 2) and 3). We regard a property of a theory (in first or second order logic) as virtuous if the property has mathematical consequences for the theory or for models of the theory. We rehearse some results of Marek Magidor, H. Friedman and Solovay to argue that for second order logic, 'categoricity' has little virtue.

For first order logic, categoricity is trivial. But 'categoricity in power' illustrates the sort of mathematical consequences we mean. One can lay out a schema with a few parameters (depending on the theory) which describes the structure of any model of any theory categorical in uncountable power. Similar schema for the decomposition of models apply to other theories according to properties defining the stability hierarchy. We describe arguments using properties, which essentially involve formalizing mathematics, to obtain results in 'core' mathematics. Further these methods (i.e. the stability hierarchy) provide an organization for much mathematics which more than fulfills a dream of Bourbaki.

• Dirk Schlimm (Dept of Philosophy, McGill University), "Axioms in mathematical practice"

Summary

In this talk I will discuss various dimensions of axioms that play a role in mathematical activities. In part, the power of axiom systems stems from the possibility of changing our perspective and using them in different ways. Thus, the same axioms can play semantic or syntactic roles, and can be intended to be descriptive or prescriptive. Putting forward an axiomatization does not commit one to one particular perspective. This becomes especially clear when looking at the origins of axioms, where a conceptual analysis of one or more domains, reasoning from theorems, and manipulation of axioms are often all present together. Thus, analyses that argue for one particular role of axioms can only be regarded as idealizations that do not capture the richness of mathematical practice. Similarly, the view according to which axiomatization is a purely cosmetic and expository enterprise, highlights a particular use of axioms, but fails to do justice to the creative role that they can play in the development of new mathematics. I will also discuss various criteria that can be, and have been, employed in assessing a system of axioms and conclude that there must be trade-offs, because more often than not these criteria stand in conflict with each other. This is one reason why axiomatizing a theory is not as straightforward

a matter as some would like it to be and why there is no single set of criteria for what makes a good axiomatization. Nevertheless, there are advantages of axiomatic presentations, like the separation of theory and reasoning, theory demarcation, internal and external systematization, and intersubjectivity, that cannot be obtained by other forms of presentations of theories. Thus, there are many reasons for employing axioms in mathematical practice that go well beyond simply providing a rigorous foundation for a discipline.

• Rebecca Morris (Dept of Philosophy, Carnegie Mellon University) (joint work with Jeremy Avigad), "Character and Object"

Summary

In 1837, Dirichlet proved that there are infinitely many primes in any arithmetic progression in which the terms do not all have a common factor. We survey implicit and explicit uses of "Dirichlet characters" in presentations of Dirichlet's proof in the nineteenth and early twentieth centuries, with an eye towards understanding some of the pragmatic pressures that shaped the evolution of modern mathematical method. We also discuss similar pressures evident in Frege's treatment of functions, and the nature of mathematical objects.

Session II

• Marianna Antonutti Marfori (Dept of Philosophy, University of Bristol) (joint work with Leon Horsten), "Human Effective Computability and Absolute Undecidability"

Summary

Kreisel's notion of human effective computability is analysed. The connection between human effective computability and absolute unde-

cidability is also explored.

• Katherine Dunlop (Dept of Philosophy, University of Texas–Austin), "Poincarés Opposition to Logicism in Arithmetic"

Summary

Poincaré's opposition to logicism in arithmetic is usually thought to rest on the Kantian view that arithmetical knowledge is based on an *a priori* intuition of natural number. On such a view, logicist reduction is at best superfluous, because we already have direct epistemic access to the subject-matter that it purports to fix by definitions. Poincaré famously argues that logicism is, worse, circular, because it assumes the principle of mathematical induction. As is well-known, the Kantian view of arithmetic is at odds with Poincaré's philosophy of geometry; I find it is also incongruous with his remarks about mathematics as a whole and scientific knowledge in general. I outline a more consonant interpretation, on which the branches of mathematics alike involve the imposition of rules at the prompting of experience, and seek to explain by its means Poincaré's opposition to logicism. I agree that Poincaré rejects logicism's account of arithmetics content or subject-matter, but my interpretation requires a different understanding of his notion of intuition.

• Ken Manders (Dept. of Philosophy, University of Pittsburgh), "Mathematical representation and the philosophy of mathematics"

Summary

Current practice in the philosophy of mathematics tends to treat mathematical representation on the paradigm of notational variants: differences are philosophically inessential, and representations may be redone rather freely for philosophical purposes.

The talk aims to articulate a philosophical picture of contributions of expressive means showing how (suitable) representational differences can matter philosophically. Such differences can be brought out by case study methodologies; there are many examples.

• Christ Menzel (Dept of Philosophy, Texas A&M University), "Wide Sets, ZFCU and the Iterative Conception"

Summary:

$ZFCU$ is ZFC modified to allow for the existence of urelements. Intuitively, it is conceivable that

M: There are more urelements than can be numbered by any cardinal.

(Indeed, Nolan and Sider have both argued quite persuasively that M is a consequence of certain modal ontologies.)

It is of course a theorem of $ZFCU$ that, given M, the proposition

S: There is a set of all urelements

is false. By contrast, on the face of it, the iterative conception of set seems entirely consistent with S, irrespective of M. For the key intuitive idea underlying the iterative conception is that sets are "formed" in a series of "stages" from an initial stock of atoms, and the sets are exactly those collections that are formed at some stage from the objects in preceding stages. Intuitively, then, the set of all urelements is formed at the very first stage—even if M is true. But this is a bit puzzling, as the iterative conception is typically thought to provide the motivation for the intended models of ZFC. In this paper I diagnose the source of this apparent disconnect and, focusing first on Replacement and then on Powerset, I develop and defend two modifications of $ZFCU$ that are consistent with the conjunction of M and S and, hence, more generally, with the existence of "wide" sets—sets which, like the sets of $ZFCU$, have a definite rank but which, like proper

classes, are too large to have a definite cardinality. I argue that both preserve a robust iterative conception of set. I conclude by proving the consistency of both theories relative to ZFC + “There exists an inaccessible cardinal”.

Session III

• Pat Reeder (Dept of Philosophy, The Ohio State University), “A ‘Non-standard Analysis’ of Euler’s *Introductio in Analysin Infinitorum*”

Summary

In Leonhard Euler’s seminal work *Introductio in Analysin Infinitorum* (1748), he readily used infinite numbers and infinitesimals in many of his proofs. In this presentation, I aim to reformulate a pair of proofs from the *Introductio* using concepts and techniques from Abraham Robinson’s celebrated non-standard analysis (NSA). I will specifically examine Euler’s proof of the Euler formula and his proof of the divergence of the harmonic series. Both of these results have been proved in subsequent centuries using epsilontic (standard epsilon-delta) arguments. The epsilontic arguments differ significantly from Euler’s original proofs. I will compare and contrast the epsilontic proofs with those I have developed by following Euler more closely through NSA. I claim that NSA possesses the tools to provide appropriate proxies of the inferential moves found in the Introductio. With the remaining time, I will offer some preliminary discussion of the purity of the methods behind the proofs. Most notably, the theory behind NSA is conservative over the theory behind ordinary analysis (in effect, due to the crucial Transfer Principle of NSA.) This peculiar feature of NSA raises special questions regarding purity. Does the use of ideal elements

count as impure when the theory that includes the ideal elements is conservative over the theory without ideal elements? Do these methods capture the letter of purity even if they do not capture the spirit of purity? These and closely related questions will be considered.

• Roy Cook (Dept of Philosophy, University of Minnesota–Twin Cities) (joint work with Philip Ebert), "Frege's Recipe"

Summary

Thanks to the work of Heck, Boolos, and others, it is widely recognized that Basic Law V plays a rather minimal, and mostly eliminable, role in the derivations of arithmetic in the *Grundgesetze*, although the philosophical ramifications of this are less well-understood. What has been less widely recognized is that Frege never explicitly proves Hume's Principle in the *Grundgesetze*. These facts seem to be in tension with (if they are not outright inconsistent with) the standard story regarding Frege's philosophy of mathematics based on the *Grundlagen*. In this paper we present a new interpretation of Frege's mature philosophy of mathematics—one that (1) stems from a careful examination of Frege's methodology for defining mathematical objects within the *Grundgesetze*, (2) helps to explain the initially odd-looking facts mentioned above, and (3) entails that Frege's views on arithmetic evolved considerably between the *Grundlagen* and *Grundgesetze*.

• Aldo Antonelli (Dept of Philosophy, University of California-Davis), "On the General Interpretation of First-Order Quantifiers"

Summary

In his 1950 dissertation, Leon Henkin showed how to provide higher-order quantifiers with non-standard, or "general" interpretations, in which, for instance, second-order quantifiers are taken to range over

collections of subsets of the domain that may fall short of the full power-set. In contrast, first-order quantifiers are usually regarded as immune to this sort of non-standard interpretations, at least in the sense that the semantics for first-order quantifiers is ordinarily taken to be determined once a first-order domain of objects is fixed.

According to the modern theory of generalized quantifiers, a first-order quantifier is construed as a predicate of subsets of the domain. For example, the first-order existential quantifier is taken to denote the collection of all non-empty subsets, the quantifier "there are exactly k" is taken to denote the collection of all k-membered subsets, etc. But the generalized conception still views first-order quantifiers as predicates over the full power-set, while the possibility that they, similarly to their second-order counterparts, might denote arbitrary collections of subsets has gone mostly unnoticed.

This talk introduces the notion of a general interpretation for arbitrary first-order quantifiers, exploring some of the properties of quantifiers so construed (especially as regards the unary case) and emphasizing the effects of imposing various further constraints that the interpretation is to satisfy. Such constraints typically come in the form of closure conditions that the second-order domain is assumed to satisfy. Among other results, we show by a model-theoretic argument that for certain closure conditions the notion of validity relative to models satisfying the conditions is axiomatizable.

Finally, although the talk is mainly devoted to laying the technical groundwork, we touch upon some of the philosophical insights that can be gained from the consideration of non-standard interpretations, especially as regards issues of semantic determinacy of first-order quantifiers and their role in expressing existence claims and ontological commitment.